AP Calculus AB | Homework 0226 | Name:
Dr. Paul L. Bailey | Wednesday, February 26, 2025

Due Thursday, February 27, 2025.

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

Definition 1. Define the *natural logarithm* to be the function

$$\log : (0, \infty) \to \mathbb{R} \quad \text{given by} \quad \log(x) = \int_1^x \frac{1}{t}\, dt.$$

We have shown that log is bijective.

Definition 2. Define the *natural exponential function* to be the inverse of the natural logarithm. Thus

$$\exp : \mathbb{R} \to (0, \infty) \quad \text{such that} \quad \exp(x) = y \Leftrightarrow x = \log(y).$$

Define the number e by

$$e = \exp(1).$$

For $a \in (0, \infty) \setminus \{1\}$ and $x \in \mathbb{R}$, define

$$a^x = \exp(x \log(a)) \quad \text{so that } e^x = \exp(x).$$

We have shown that $\frac{d}{dx} \exp(x) = \exp(x)$.

Problem 1 (Thomas §7.2 # 13)**.** Find $\frac{dy}{dx}$ where

$$y = \ln x^3.$$

Problem 2 (Thomas §7.2 # 21)**.** Find $\frac{dy}{dx}$ where

$$y = \frac{\ln x}{1 + \ln x}.$$

Problem 3 (Thomas §7.2 # 45)**.** Compute

$$\int_2^4 \frac{dx}{x(\ln x)^2}.$$

Problem 4 (Thomas §3.5 # 35)**.** Find $\frac{dr}{d\theta}$ where

$$r = \sin(\theta^2)\cos(2\theta).$$

Problem 5 (Thomas §3.6 # 25)**.** Find $\frac{dy}{dx}$ where

$$y^2 = \frac{x-1}{x+1}.$$

Problem 6 (Thomas §5.5 # 35)**.** Compute

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\, dt.$$

Problem 7 (Thomas §3.5 # 60)**.** Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	-1/3	-8/3

Find the derivatives with respect to x of the following combinations at the given value of x.

(a) $5f(x) - g(x)$, $x = 1$

(b) $f(x)g^3(x)$, $x = 0$

(c) $\dfrac{f(x)}{g(x)+1}$, $x = 1$

(d) $f(g(x))$, $x = 0$

(e) $g(f(x))$, $x = 0$

(f) $(x^{11} + f(x))^{-2}$, $x = 1$

(g) $f(x + g(x))$, $x = 0$

Problem 8. Let

$$f(x) = 6x^3 - 11x^2 - 24x + 9.$$

Note that $f(3) = 0$. Find all zeros of f.

Problem 9. Consider the family of functions $f(x) = x^4 - ax^2$. Show that f has a local maximum if and only if f has 3 distinct zeros.

Problem 10 (Thomas Problem §4.5 # 12)**.** Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.